A certified de Bruijn–Newman upper bound of 0.1729

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Abstract

In the standard Polymath15 normalization, the de Bruijn–Newman constant satisfies $\Lambda \leq 1729/10000 = 0.1729$. The certificate combines the published verified zeta-zero height, a cutoff-uniform sparse four-prime finite canopy, complementary analytic tails, and an 844-rectangle argument-principle barrier. It improves the established bound $\Lambda \leq 11/50 = 0.22$ by exactly $471/10000 = 0.0471$ and replaces the earlier under-audit 0.175 target.

$$\Lambda \leq \frac{1729}{10000} = 0.1729$$

1 The statement

Let H_t and Λ have the normalization of the pinned Polymath15 source. Set $X = 6000000185827$, $t_0 = 3377/20000$, and $y_0 = 9/100$. Polymath15 Theorem 1.2 converts the three certified zero-free regions into the displayed bound.

Relative to the established Polymath15 theorem $\Lambda \leq 11/50$, the exact decrease is $471/10000 = 0.0471$.

$$\Lambda \leq t_0 + \frac{1}{2}y_0^2 = \frac{1729}{10000} = 0.1729$$

2 The three zero-free regions

The initial-time strip uses the published S-0001 theorem only for critical-line location through height 3000175332800. The required heights $X/2$ and $(X+1)/2$ leave reserves 175239886.5 and 175239886; no zero-simplicity input is used.

At $t = t_0$, 1,649 low-y cells and 107 high-y cells cover the finite ray. Four Euler primes are used through $N = 770000$, three through $N = 2000000$, and two thereafter, followed by complementary analytic tails.

The intermediate barrier is covered by a pinned V3 stored-sum matrix and 844 adaptive argument-principle rectangles. The winding enclosure contains only the integer zero, and the exact-model boundary margin exceeds 0.95116.

3 The cutoff repair

The sparse checker evaluates the literal state at the initial cutoff and after every reachable divisor threshold before taking termwise maxima. This repairs the terminal-cutoff substitution defect identified by O-0177.

The package-native regression enumerates all 451 reachable zero-through-four-prime state patterns, including all 353 four-prime patterns. On an actual theorem cell it rejects an omitted-initial-state counterfeit that passes the older bounded regression, closing the fail-open boundary identified by O-0184.

$$N_1 = 690988, \quad N_2 = 692000, \quad r = 691110, \quad \text{states } (32767, 65535)$$

4 Independent finite evidence

Local arm64 and decorrelated x86 FLINT 3.6.0 evidence cover all 1,756 finite cells in canonical order. The weakest low-y and high-y endpoints are $0.0016881143766\ldots$ and $0.0241907009721\ldots$, both above the raw threshold 0.00114.

The complete effective-model transfer cost is at most $0.00113438368318\ldots$, leaving strict positive reserve. The default verifier authenticates the package, reruns every analytic seam and state-pattern gate, checks both architecture evidence layers, and fails closed on source, runtime, grid, matrix, or transcript drift.

5 Pinned certificate

The pinned verifier authenticates the full package inventory, theorem and S-0001 source layers, all local and x86 finite-cell evidence, the parameter and error-transfer calculations, both analytic tails, the complete four-prime pattern-space regression, scalar-versus-polynomial parity, counterfeit controls, and a semantic V3 matrix read. The release-strength full-barrier mode also reruns and byte-compares all 844 argument-principle rectangles.

Run command

```
nice -n 19 uv run --frozen python canon/witnesses/C-0130/verify.py
```

Pinned files

- `canon/witnesses/C-0130/verify.py`
- `canon/witnesses/C-0130/PROOF.md`
- `canon/witnesses/C-0130/PIN.md`
- `canon/witnesses/C-0130/DEPENDENCIES.md`

- `canon/witnesses/C-0130/MANIFEST.sha256`

6 Scope

The certificate proves $\Lambda \leq 1729/10000 = 0.1729$ in the standard Polymath15 normalization. It does not claim optimality, and it gives no lower bound on Λ.

7 Sources

- Canonical claim: `canon/claims/C-0130-certified-debruijn-newman-upper-bound-01729.md`
- Proof: `canon/witnesses/C-0130/PROOF.md`
- Pinned witness: `canon/witnesses/C-0130/PIN.md`
- Published theorem source: D.H.J. Polymath, Effective approximation of heat flow evolution of the Riemann xi function, and a new upper bound for the de Bruijn–Newman constant, Res. Math. Sci. 6 (2019), article 31. `https://doi.org/10.1007/s40687-019-0193-1`
- Published finite-height input: D. J. Platt and T. S. Trudgian, The Riemann hypothesis is true up to $3{\cdot}10^{12}$, Bull. Lond. Math. Soc. 53 (2021), 792–797. `https://doi.org/10.1112/blms.12460`