Bellotti–Wong's six-parameter zero-count pair sharpens exactly

Kenan

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Abstract

Bellotti–Wong's printed six-parameter tuple gives the exact leading coefficient 0.176616, rather than the outward decimal 0.1767. Retaining the source normalization $\log(T/(2\pi))$ also converts the additive constant 15.805 to $15.4804015\ldots$. The resulting pair applies on the source's $T \geq 1$ domains for finite-order Hecke, corrected-conductor Dirichlet, Dedekind, and Riemann zero counts, and conditionally for Artin L-functions.

$$(C_1, D) = \left(\frac{22077}{125000}, \frac{3161}{200} - \frac{22077}{125000}\log(2\pi)\right) = (0.176616, 15.4804015\ldots)$$

1 The sharpened pair

Set $c = 22077/125000 = 0.176616$ and $D = 3161/200 - c\log(2\pi)$. The full-plane zero-count error has leading coefficient c and additive coefficient D, with $15.4804 < D < 15.4805$.

Both coefficients are strictly below Bellotti–Wong's published six-parameter Table 6 pair $(0.1767, 15.805)$ on the identical $T \geq 1$ range.

$$|N(T, \chi) - M_\chi(T)| \leq c\big(\log A(\chi) + m_\chi \log T\big) + Dm_\chi$$

2 Exact leading coefficient

The source's symmetric zero-count construction has $C_1 = da_1/4$. Its printed six-parameter tuple gives $d = 18/25$ and $a_1 = 2453/2500$, so the exact product is $22077/125000$.

The additional higher-derivative operators in the six-parameter construction do not alter the conductor coefficient: higher derivatives of the logarithmic derivative of A^s vanish, while their gamma contributions remain in the bounded remainder represented by C_2.

$$C_1 = \frac{18}{25}\frac{2453}{2500}\frac{1}{4} = \frac{22077}{125000}$$

$$\frac{1767}{10000} - \frac{22077}{125000} = \frac{21}{250000} > 0$$

3 Retaining the normalization

Table 6 supplies $C_2 = 15.805$ in a formula containing $\log(T/(2\pi))$, not $\log T$. Expanding that logarithm preserves the exact leading coefficient and subtracts $c\log(2\pi)$ from the additive term.

The improvement is therefore an exact normalization consequence of the certified source tuple and Table 6 constant, not a new numerical optimization of the six-parameter admissibility problem.

$$c\log\frac{T}{2\pi} + \frac{3161}{200} = c\log T + \left(\frac{3161}{200} - c\log(2\pi)\right)$$

1. Read the exact rational values behind the printed six-parameter tuple.
2. Compute the source coefficient $C_1 = da_1/4$ exactly.
3. Keep the source's $\log(T/(2\pi))$ normalization when rewriting the bound in conductor form.

4 Specializations and Pareto position

The same pair applies unconditionally to nontrivial finite-order Hecke L-functions. For imprimitive Dirichlet characters, the conductor is that of the inducing primitive character because the added Euler-factor zeros lie on $\Re s = 0$, outside the counted strip.

The Dedekind bound retains the source constant 2.033. Exact halving under the source's upper-half Riemann convention gives the pair $(0.088308, 8.7567007520\ldots)$. This has a smaller leading but larger additive coefficient than C-0082, so it is an additional Pareto point rather than a uniform replacement.

$$|N_K(T) - M_K(T)| \le c(\log d_K + n_K \log T) + Dn_K + 2.033$$

$$\left|N(T) - \frac{T}{2\pi}\log\frac{T}{2\pi e}\right| \le 0.088308\log T + 8.7567007520\ldots$$

5 What the certificate establishes

The verifier hash-pins the Bellotti–Wong v1 PDF, source archive, extracted TeX, metadata, and the C-0082 dependency. It checks the literal source anchors for the symmetric normalization, the exact coefficient formula, the printed tuple, Table 6, the source's certification statement, and the exploratory eight-parameter caveat.

Exact rational arithmetic and 256-bit Arb then certify the coefficient gap, both additive constants, exact Riemann halving, conductor correction, and the Pareto comparison. Counterfeit checks reject mixing the rounded coefficient with the exact subtraction, omitting half the Dedekind constant, or retaining the ambient Dirichlet modulus.

6 Pinned certificate

The certificate pins the Bellotti–Wong v1 source and C-0082 conventions, derives the exact coefficient and retained-normalization constant, propagates them through the Hecke, Dirichlet, Dedekind, Riemann, and conditional Artin forms, and rejects normalization and count-convention counterfeits.

Run command

```
uv run --frozen python canon/witnesses/C-0103/verify.py
```

Pinned files

- `canon/witnesses/C-0103/verify.py`
- `canon/witnesses/C-0103/counterfeit.py`
- `canon/witnesses/C-0103/PROOF.md`
- `canon/witnesses/C-0103/PIN.md`
- `canon/witnesses/C-0082/source/arxiv/version_june_2026.tex`
- `canon/claims/C-0082-bellotti-wong-zero-count-retained-log2pi.md`

7 Scope

The calculation retains Bellotti–Wong's certified six-parameter tuple and source normalization. The Artin L-function branch assumes Artin holomorphy.

8 Sources

- Canonical claim: `canon/claims/C-0103-bellotti-wong-six-parameter-exact-zero-count-pareto.md`
- Proof: `canon/witnesses/C-0103/PROOF.md`
- Source and verifier pin: `canon/witnesses/C-0103/PIN.md`
- Pinned Bellotti–Wong source: `canon/witnesses/C-0082/source/arxiv/version_june_2026.tex`
- Count-convention dependency: `canon/claims/C-0082-bellotti-wong-zero-count-retained-log2pi.md`