Johnston–Yang's surviving global VK psi decay improves to 0.2043325

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Abstract

The global explicit bound for the Chebyshev function keeps Johnston–Yang's amplitude 0.026, logarithmic power 1.801, and full range $x \geq 23$, while increasing the Vinogradov–Korobov decay constant to 0.2043325. The proof uses only Yang's surviving zero-free denominator 51.34, interiorized to 51.3401, then reoptimizes the large-x tail and closes the remaining ranges with finite, Buethe, and C-0067 splices.

$$|\psi(x) - x| < 0.026x(\log x)^{1.801} \exp\left(-0.2043325 \frac{(\log x)^{3/5}}{(\log\log x)^{1/5}}\right) \quad (x \geq 23)$$

1 The improved envelope

For $L = \log x$ and $r(L) = L^{3/5}/(\log L)^{1/5}$, the claimed envelope holds for every real $x \geq 23$. It improves the published decay 0.1853 and the previous certified same-form decay 0.2043 without changing any other headline parameter.

Because the amplitude, power, and domain are identical, the larger decay constant makes the new right-hand side strictly smaller at every point of the common range.

$$|\psi(x) - x| < 0.026xL^{1.801}e^{-0.2043325r(L)}$$

$$0.2043325 - 0.1853 = \frac{7613}{400000}, \qquad 0.2043325 - 0.2043 = \frac{13}{400000}$$

2 The surviving zero-free input

Yang's theorem gives an open Vinogradov–Korobov zero-free region with denominator 51.34. Replacing it by the closed denominator $c_* = 51.3401$ moves the boundary strictly inside that open region and introduces an exact padding of 10^{-4}.

Minimizing the Johnston–Yang height expression then gives a lower coefficient $Q(c_*) = 0.2043529808\ldots$, which is safely above the target decay. No input from the retracted 51.323 branch is used.

$$\zeta(\sigma + it) \neq 0 \quad \left(\sigma > 1 - \frac{1}{51.34(\log t)^{2/3}(\log\log t)^{1/3}}\right)$$

$$Q(c) = \left(\frac{5}{3c^3}\right)^{1/5}\left[\left(\frac{3}{2}\right)^{2/5} + \left(\frac{2}{3}\right)^{3/5}\right]$$

3 Large-x tail

The proof chooses $L_0 = 950000000$, $\sigma = 0.9999714$, $\theta = 0.08917$, and $B_3 = 0.21617$. Directed interpolation of the source density rows supplies the associated constants, while the fixed trial proves $Q(c_*)r(L) \leq \log T \leq B_3 r(L)$ for every $L \geq L_0$.

The repaired explicit-formula terms are normalized by the target envelope. Each normalized contribution is proved decreasing from L_0 onward, and their endpoint sum is below one with reserve about 0.0501. The native first-density decay is $0.2043373955\ldots$, still strictly above 0.2043325.

$$\frac{s_1 + s_2 + s_3}{0.026L^{1.801}e^{-0.2043325r(L)}} \leq 0.9498732528858\ldots < 1 \qquad (L \geq L_0)$$

1. Interiorize the open zero-free denominator and derive the coefficient $Q(c_*)$.
2. Choose a source-admissible density tuple and prove the upper and lower height bounds uniformly from L_0.
3. Normalize every explicit-formula term by the target envelope and reduce the tail to endpoint inequalities plus monotonicity.

4 Closing the full range

For $23 \leq x \leq 59$, the verifier uses that $\psi(x)$ is constant between prime powers and checks both endpoints of every interval. From 59 to e^{58} it compares against Johnston–Yang's explicit Buethe bound.

For $58 \leq L \leq L_0$, the target is compared with C-0067's global FKS–Yang bound. An analytic derivative reduction shows that the logarithmic comparison has no interior minimum, so positive endpoint margins cover the entire middle interval.

$$\frac{|\psi(x) - x|}{x} < 9.2202181L^{3/2}e^{-0.88178\sqrt{L}}$$

5 What the certificate establishes

The directed 256-bit Arb verifier hash-pins the Johnston–Yang source, Yang's thesis, C-0067, C-0070, and a forward use of the published bound. It checks the source interpolation, the open-to-closed padding, the exact $Q(c)$ calculation, every tail derivative, every range splice, and both exact improvements.

It also reruns the repaired published 0.1853 theorem, including the printed B_2 defect, so the comparison baseline is reproduced rather than assumed. A separately pinned independent checker is corroborative; the interval bounds and analytic monotonicity reductions are the proof.

6 Pinned certificate

The public verifier certifies the source pins, the surviving zero-free input, the reoptimized large-x tail, all finite and middle-range splices, and the exact comparison with both prior same-form bounds. The independent checker is an additional corroborative artifact, not the logical basis of the theorem.

Run command

```
uv run --frozen python canon/witnesses/C-0099/verify.py
```

Pinned files

- `canon/witnesses/C-0099/verify.py`
- `canon/witnesses/C-0099/independent_check.py`
- `canon/witnesses/C-0099/PROOF.md`
- `canon/witnesses/C-0099/PIN.md`

7 Scope

The global bound keeps amplitude 0.026, logarithmic power 1.801, and range $x \geq 23$, using the denominator 51.34 closed inward to 51.3401.

8 Sources

- Canonical claim: `canon/claims/C-0099-johnston-yang-surviving-global-vk-psi-decay-0-2043325.md`
- Proof: `canon/witnesses/C-0099/PROOF.md`
- Source and verifier pin: `canon/witnesses/C-0099/PIN.md`