

Fiori–Kadiri–Swidinsky’s global absolute psi decay improves to 0.8817882

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Abstract

For every real $x > 2$, the global absolute Chebyshev- ψ bound holds with amplitude 9.2202181 and decay 0.8817882. This keeps the amplitude and full range of the preceding certified bound while increasing its decay from 0.88178, and it improves the published Fiori-Kadiri-Swidinsky decay 0.8476836 in the same displayed form.

$$|\psi(x) - x| < 9.2202181x(\log x)^{3/2}e^{-0.8817882\sqrt{\log x}}$$

1 What is proved

Writing $L = \log x$, the theorem gives one explicit inequality for the complete range $x > 2$. Because the amplitude is unchanged from the preceding 0.88178 result, the larger decay makes the right-hand side strictly smaller at every point in the range.

$$\frac{|\psi(x) - x|}{x} < 9.2202181L^{3/2}e^{-0.8817882\sqrt{L}} \quad (x > 2)$$

2 Supply an admissible zero-free input

The upstream classical zero-free theorem gives an open region with denominator 4.8568. Fiori-Kadiri-Swidinsky formulate their input on a closed boundary, so the proof uses the slightly larger exact denominator $R_* = 4.85681$.

That closed region lies strictly inside the proved open region. The padding is therefore a logical interiorization, not a numerical improvement of the zero-free theorem.

$$R_* = 4.85681 > 4.8568$$

$$\zeta(\sigma + it) \neq 0 \quad \text{for } \sigma > 1 - \frac{1}{4.8568 \log t}$$

3 Recompute the asymptotic envelope

At the handoff $L = 2008$, the certificate recomputes all 99 999 cells in the Fiori-Kadiri-Swidinsky amplitude formulas using R_* . Normalization produces a tail coefficient C_* and decay d_* .

Since $d_* > 0.8817882$, conversion to the target decay becomes easier as L increases. It is enough to check the tail at the handoff.

$$C_* = 28.6643229697\dots, \quad d_* = \frac{2}{\sqrt{R_*}} = 0.9075163092\dots$$

$$E_\psi(x) \leq C_* L^{3/2} e^{-d_* \sqrt{L}} \quad (L \geq 2008)$$

4 Splice the low and high ranges

For $0 < L \leq 2008$, the published low-range envelope has coefficient 2 and decay 0.8476836. Converting it to the target decay requires the largest coefficient at $L = 2008$.

For $L \geq 2008$, the recomputed asymptotic envelope requires its largest converted coefficient at the same endpoint. Directed arithmetic proves positive margins on both sides, so the two bounds splice into one all-range inequality.

$$d_{\text{low}} = 0.8476836 + \frac{\log(9.2202181/2)}{\sqrt{2008}} = 0.8817882015416\dots$$

$$9.2202181 - C_* e^{-(d_* - 0.8817882)\sqrt{2008}} > 0.1702763$$

5 Role of the certificate

The verifier hash-pins and reruns the C-0096 zero-free certificate and the C-0067 Fiori-Kadiri-Swidinsky reconstruction, including their source checks. It then restores directed precision, checks the open-to-closed padding, recomputes every amplitude cell, and certifies both splice margins.

The proof therefore covers the absolute value and every real $x > 2$ rather than only the asymptotic range.

6 Pinned certificate

The certificate reruns the pinned zero-free and prior psi-bound dependencies, interiorizes the open zero-free radius, recomputes the complete Fiori-Kadiri-Swidinsky amplitude at the new input, and proves positive directed margins for both sides of the $\log x = 2008$ splice.

Run command

```
uv run --frozen python canon/witnesses/C-0098/verify.py
```

Pinned files

- `canon/witnesses/C-0098/verify.py`
- `canon/witnesses/C-0098/PIN.md`
- `canon/witnesses/C-0096/verify.py`
- `canon/witnesses/C-0067/verify.py`
- `canon/claims/C-0096-global-classical-zeta-zero-free-denominator-4-8568.md`
- `canon/claims/C-0067-fks-yang-global-psi-decay-0-88178.md`

7 Scope

The bound holds for every real $x > 2$ with amplitude 9.2202181. The active low-range splice fixes the same-amplitude decay cap at 0.8817882015416....

8 Sources

- Canonical claim: `canon/claims/C-0098-fks-c0096-global-psi-decay-0-8817882.md`
- Certificate pin: `canon/witnesses/C-0098/PIN.md`
- Zero-free dependency: `canon/claims/C-0096-global-classical-zeta-zero-free-denominator-4-8568.md`
- Prior global psi bound: `canon/claims/C-0067-fks-yang-global-psi-decay-0-88178.md`