Strict TP4 for the folded reciprocal heat kernel on arithmetic cells

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Abstract

For $K(q,l) = l^{-5/4}e^{-q/l} + l^{5/4}e^{-ql}$, the theorem proves strict positivity of every 4×4 determinant whose row knots lie in four ordered arithmetic square cells and whose spectral knots satisfy $1 < l_1 < \cdots < l_4$. The proof reduces the determinant to oriented tangent volumes of a projective jet curve, certifies one cumulative support-line inequality across the only defective arithmetic region, propagates that inequality through an outer strictly convex Gauss curve, and then applies the Peano and extended-complete-Chebyshev transports. Positive independent row integration preserves the strict determinant on the same product-support domain.

$$q_i = \pi x_i^2 \text{ in ordered cells} \Longrightarrow \det[K(q_i, l_j)]_{i,j=1}^4 > 0$$

1 What is proved

Fix integers $1 \le n_1 < n_2 < n_3 < n_4$. Choose $x_i \in [n_i, n_i+1]$ so that $q_i = \pi x_i^2$ are strictly increasing, and choose $1 < l_1 < l_2 < l_3 < l_4$. The result is pointwise strict positivity of the kernel determinant.

The row cells are arithmetic because they arise from consecutive square coordinates $q = \pi x^2$ with x restricted to ordered unit intervals. The theorem permits arbitrary points inside the four chosen cells, subject to strict ordering.

$$K(q,l) = l^{-5/4}e^{-q/l} + l^{5/4}e^{-ql}$$

$$\det[K(q_i, l_j)]_{i,j=1}^4 > 0$$

2 Projective reduction

Let $D = l\partial_l$ and divide the first four spectral jets by the positive kernel value. This gives the affine projective curve $\gamma_l(q) = (DK/K, D^2K/K, D^3K/K)$. Its derivative factors as $\gamma_l'(q) = f_q(q,l)(1, \rho(q,l), \sigma(q,l))$, where $f_q > 0$ follows from the lower-rank theory.

Successive row differences give a projective Peano identity. Thus the rank-four coalesced-column determinant is an integral of oriented triples of tangents over the three consecutive row gaps.

Positivity of those tangent orientations on every admissible transversal is the geometric core of the proof.

$$\det[1, \gamma_l(q_i)]_{i=1}^4 = \int_{q_1}^{q_2}\int_{q_2}^{q_3}\int_{q_3}^{q_4} \det\big(\gamma_l'(s), \gamma_l'(t), \gamma_l'(u)\big)\, du\, dt\, ds$$

$$\Gamma_l(q) = (\rho, \sigma) = \left(\frac{g_q}{f_q}, \frac{h_q}{f_q}\right)$$

3 The certified support inequality

The Gauss curve is already strictly convex beyond its last rank-four wall $q_4^* = 5.3611335998\ldots < 4\pi$. The only missing configuration has one tangent in the first arithmetic region and two tangents at or beyond 4π. It is controlled by a support line at $Q = 4\pi$.

Exact elimination expresses the support determinant as a positive denominator times a 736-term numerator. The certificate proves positivity on $\pi \le q \le 2681/500$, a rational interval containing q_4^*, in three overlapping regimes: an order-six Taylor quotient near $l = 1$, an outward-rounded compact cover, and an exact rational activity-tail estimate for $l \ge 20$.

$$\det\big(\Gamma_l(4\pi) - \Gamma_l(q), \Gamma_l'(4\pi)\big) > 0$$

$$[t^6]N = \frac{32}{15}(4\pi - q)^2 P(q, \pi), \qquad l = e^t$$

4 From one support line to TP4

Write the outer Gauss curve as a strictly convex graph $\sigma = \phi(\rho)$. For a base point $P = (r_0, s_0) = \Gamma_l(q)$, the certified inequality says that the tangent at $\rho(4\pi)$ lies above the secant from P. The derivative identity $H'(r) = (r - r_0)\phi''(r) > 0$ propagates this orientation to every pair of later points on the outer curve.

Consequently every tangent triple selected from the three arithmetic row gaps has positive orientation: either all three points lie in the outer convex chamber, or the first lies below 4π and the support-line argument applies. The Peano identity gives the coalesced rank-four Wronskian. Together with the strict rank-one, rank-two, and rank-three Wronskians, the extended-complete-Chebyshev criterion transports positivity to all ordered spectral knots.

$$H(r) = (r - r_0)\phi'(r) - \phi(r) + s_0, \qquad H'(r) = (r - r_0)\phi''(r) > 0$$

$$\det[D^{j-1}K(q_i, l)]_{i,j=1}^4 > 0$$

5 Row integration and certificate role

If row i is independently integrated against a nonzero finite positive measure and the product measure is concentrated on the same ordered four-cell domain, multilinearity and Fubini express the integrated determinant as the integral of the pointwise determinant. Since the integrand is strictly positive on the product support, the integrated determinant is also strictly positive.

The pinned certificate closes the load-bearing analytic inequality: it reconstructs the normalized Gauss curve, the support numerator and positive denominator, the wall expansion, the compact Arb cover, the tail bound, the compact-tail overlap, and the two calculus identities used for support propagation. The passage from that inequality to tangent orientation, the Peano identity, ECT transport, and row integration is the exact written proof route rather than a sampled determinant test.

$$\det\left[\int K(q,l_j)\,d\mu_i(q)\right]_{i,j=1}^{4} = \int\cdots\int \det[K(q_i,l_j)]_{i,j=1}^{4}\,d\mu_1\cdots d\mu_4$$

6 Pinned certificate

The pinned verifier certifies the cumulative Gauss support inequality on the full wall, compact, and tail domains and checks the exact symbolic identities needed to propagate that support line. This inequality is the analytic input that, combined with the written convexity, Peano, ECT, and integration arguments, proves the stated arithmetic-cell TP4 determinant.

Run command

```
uv run --with sympy --with python-flint python canon/witnesses/C-0036/verify.py
```

Pinned files

- `canon/witnesses/C-0036/PIN.md`
- `canon/witnesses/C-0036/verify.py`

7 Scope

Strict TP4 is established for row knots in four ordered arithmetic square cells, spectral knots $1 < l_1 < \cdots < l_4$, and the stated positive row integrations.

8 Sources

- Canonical claim: `canon/claims/C-0036-folded-reciprocal-heat-kernel-arithmetic-tp4.md`

- Witness pin: `canon/witnesses/C-0036/PIN.md`
- Certified support result: `scratch/adjacent-unconditional--folded-density-rank-four-gauss-support/ RESULT.md`
- Projective Peano and barycentric reduction: `archive/notes/284-arithmetic-curvature-matroid-and-rank md`
- Reciprocal tangent-sector reduction: `archive/notes/286-reciprocal-sector-reduction-for-the-rank-fo md`
- Arithmetic corner compensation: `archive/notes/300-complete-rank-four-arithmetic-barycentric-comp md`
- Pinned proof source: `scratch/adjacent-unconditional--folded-density-rank-four-gauss-support/ verify_support.py`