The reciprocal-xi Fourier transform is strictly PF7

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Abstract

Let F be the Fourier transform of $1/\xi(1/2+x)$ and put $\Lambda = F/(2\pi)$. The proof certifies a complete sequence of signed confluent Hankel determinants $H_1, \ldots, H_7$ as strictly positive for every real argument. Exact rational compact covers handle bounded t, while contour residues, derivative bridges, and eventual packet dominance control the tails. A pinned extended-complete-Chebyshev collocation theorem then promotes the complete confluent flag to strict positivity of every translation minor of order at most seven for strictly ordered row and column nodes.

$$\det[\Lambda(x_i - y_j)]_{i,j=1}^{r} > 0 \quad (1 \le r \le 7;\ x_1 < \cdots < x_r,\ y_1 < \cdots < y_r)$$

1 What is proved

Define $F(t) = \int_{\mathbf{R}} e^{itx}/\xi(1/2+x)\,dx$ and $\Lambda(t) = F(t)/(2\pi)$. For each m, the signed confluent determinant H_m is formed from derivatives of F.

The theorem proves $H_m(t) > 0$ for every real t and every $1 \le m \le 7$. Consequently every order-r translation minor of Λ is strictly positive when $1 \le r \le 7$ and both node lists are strictly increasing.

$$H_m(t) = (-1)^{m(m-1)/2} \det[F^{(i+j)}(t)]_{i,j=0}^{m-1} > 0$$

$$\det[\Lambda(x_i - y_j)]_{i,j=1}^{r} > 0 \qquad (1 \le r \le 7)$$

2 Why the complete flag matters

A top determinant alone is not enough. The collocation step requires every initial Wronskian $H_1, \ldots, H_r$ to have the correct strict sign.

For fixed x, differentiating $F(x-y)$ in y contributes exactly the sign $(-1)^{k(k-1)/2}$ appearing in H_k. A first collocation argument gives positive derivative matrices in the y nodes; a second argument in the x nodes uses their transpose. Scaling from F to Λ contributes only the positive factor $(2\pi)^{-r}$.

$$W_y(F(x-y), F'(x-y), \ldots, F^{(k-1)}(x-y)) = H_k(x-y)$$

3 Compact certification

For the seventh determinant, the bounded interval $0 \le t \le 7/5$ is covered by 432 exact rational cells. Midpoint LDL^T congruences and a degree-29 truncated-series determinant produce a positive lower bound on every cell.

The minimum transformed lower bound is about 0.3024160689, safely above zero. The inherited lower-order certificates provide their own compact covers and margins, so the complete flag through order seven is retained.

$$\min_{0\le t\le 7/5} H_7(t) \text{ is certified through a lower bound } 0.30241606892478685590\ldots > 0$$

4 Tail certification

For $t \ge 7/5$, the contour is shifted to height 100 and crosses 29 certified simple critical-line zeros. Aggregate determinant formulas cover all $\binom{29}{7} = 1560780$ pole packets without listing them one by one.

At the splice, the pole contribution exceeds the contour error by about 0.0201613375. A 40-cell derivative bridge reaches $t = 2$, after which a complete adverse-packet ratio below one makes the favorable term remain dominant. Together with the symmetry of the setup, the compact and tail arguments cover the full real line.

$$0.02071121274927569047 4\ldots - 0.00054987520791071362648\ldots > 0$$

$$0.62460118855714591283\ldots < 1$$

5 What the certificate checks

The pinned audit checks every retained PF7 row and proof identity, including the exact compact cover, rational congruences, residue and contour totals, derivative cells, splice arithmetic, and eventual packet reconstruction. The zeros crossed by the contour are justified by Platt and Trudgian's published finite-height verification.

The initial flag through PF6 is pinned through its producer and result graph. The audit independently parses the retained lower-order covers, ratios, decompositions, residues, splices, and hostile branches, but some quadrature-produced moment and first-tail enclosures remain hash-pinned primary certificate inputs rather than newly recomputed integrals. Counterfeit modes test weighted tails, reported summaries, and eventual tuples.

6 Pinned certificate

The certificate audits the complete PF1-through-PF7 flag, the compact and contour-tail proof for H_7, and the two-step collocation promotion from confluent determinants to strict translation minors. Platt and Trudgian's published finite-height theorem is used as a cited premise.

Run command

```
uv run --frozen python canon/witnesses/C-0023/verify.py
```

Pinned files

- `canon/witnesses/C-0023/PIN.md`
- `canon/witnesses/C-0023/audit.py`
- `canon/witnesses/C-0023/verify.py`
- `canon/witnesses/C-0023/source/AUDIT.md`
- `audits/2026-07-19-c0023-pf7-certificate-audit.md`
- `canon/claims/C-0023-reciprocal-xi-transform-is-strict-pf7.md`

7 Scope

Strict positivity is certified for translation minors of orders 1 through 7 at strictly ordered distinct nodes. Coalesced or weakly ordered nodes receive the nonnegative limiting statement.

8 Sources

- Canonical claim: `canon/claims/C-0023-reciprocal-xi-transform-is-strict-pf7.md`
- Witness pin: `canon/witnesses/C-0023/PIN.md`
- Collocation source audit: `canon/witnesses/C-0023/source/AUDIT.md`
- Certificate audit: `audits/2026-07-19-c0023-pf7-certificate-audit.md`
- Published finite-height theorem: D. J. Platt and T. S. Trudgian, "The Riemann hypothesis is true up to $3 \cdot 10^{12}$," Bulletin of the London Mathematical Society 53 (2021), 792–797. `https://doi.org/10.1112/blms.12460`