Strict reflected Lorentz cones for Rankin coefficient packets

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Abstract

Two exact standard Arthur parameters for O_{24} share a common [21] block. Cancelling that block compares a split degree-three Euler packet with the symmetric-square packet attached to Ramanujan's Δ. Deligne's bound makes the compact packet uniformly small relative to the split term, giving strict reflected-Lorentz inequalities for every coefficient index $n > 1$ with explicit reserves. Removing the quadratic invariant and applying exact local Rankin identities yields a second family of strict inequalities for the corresponding harmonic coefficient packets.

$$0 < D_n < 2R_n, \qquad A_n^2 > B_n^2 \quad (n > 1)$$

1 What is proved

Let $e(n)$ be the Dirichlet coefficient of $Z_3(s) = \zeta(s-11)\zeta(s)\zeta(s+11)$ and let $r(n)$ be the coefficient of $L(s, \mathrm{Ad}\,\Delta)$. With the zero-weight count $z(n)$, define $D_n = e(n)-r(n)$ and $R_n = e(n)+r(n)-2z(n)$.

For every integer $n > 1$, the coefficient packet lies strictly inside the reflected Lorentz cone: $0 < D_n < 2R_n$. The proof also gives explicit positive lower reserves and a harmonic version after division by $\zeta(2s)$.

$$0 < D_n < 2R_n \qquad (n > 1)$$

$$A_n^2 - B_n^2 > 0, \qquad A_n^2 > 4d(n), \qquad A_n^2 + 3B_n^2 - 4d(n) > 0$$

2 Where the packets come from

The pinned Chenevier–Lannes table gives the first two standard parameters as $[23] \oplus [1]$ and $\mathrm{Sym}^2\,\Delta_{11} \oplus [21]$. Their shift convention expands a block $\pi[d]$ into symmetrically shifted L-functions.

Cancelling the common [21] block leaves the split shifts $-11, 0, 11$ on one side and the adjoint symmetric-square L-function on the other. This source normalization is part of the theorem, not a heuristic analogy between Euler factors.

$$[23] \oplus [1] \quad \text{versus} \quad \mathrm{Sym}^2 \Delta_{11} \oplus [21]$$

$$Z_3(s) = \zeta(s-11)\zeta(s)\zeta(s+11)$$

3 The coefficient bounds

At a prime p, the split local block has eigenvalues $p^{11}, 1, p^{-11}$. The normalized symmetric-square block has three unit-modulus eigenvalues because Deligne's theorem gives the Ramanujan bound for Δ.

The degree-k Euler coefficient is a symmetric-power character. The split coefficient contains a term at least p^{11k}, while the compact coefficient has absolute value at most $\binom{k+2}{2}$. Multiplicativity turns these local estimates into all-n bounds.

$$e(n) \geq n^{11}, \qquad |r(n)| \leq h_3(n), \qquad h_3(n) = \prod_{p^k \| n} \binom{k+2}{2}$$

4 Strict reserves and harmonic reduction

If $z(n) = \prod_{p^k \| n} (\lfloor k/2 \rfloor + 1)$, the proof obtains explicit reserves for D_n, R_n, and $2R_n - D_n$. The elementary comparison $n^{11} \geq 2048^{\Omega(n)}$ dominates the character and zero-weight terms for every $n > 1$.

Multiplying each local Euler factor by $1 - p^{-2s}$ removes the quadratic invariant and leaves one copy of each harmonic weight. Exact local Rankin identities then identify the coefficients with squares of $A_n = \sigma_{11}(n)/n^{11/2}$ and $B_n = \tau(n)/n^{11/2}$, from which the harmonic cone inequalities follow.

$$D_n \geq n^{11} - h_3(n) > 0$$

$$R_n \geq n^{11} - h_3(n) - 2z(n) > 0$$

$$2R_n - D_n \geq n^{11} - 3h_3(n) - 4z(n) > 0$$

5 What the certificate checks

The certificate pins the Chenevier–Lannes parameter table and shift convention together with Deligne's purity-to-Ramanujan source chain. It reconstructs the [21] cancellation, the symmetric-square trace, the complete and harmonic character identities, and the factorization-sensitive all-n inequalities.

Finite exact scans are used only as mutation guards and fixtures; the universal statement comes from the multiplicative proof. The verifier emits the positive $n = 2$ margins and rejects a nearby counterfeit in which the common block is changed to [19], because the source-normalization gate then leaves the wrong shifts.

$$D_2 = \frac{4197825}{2048}, \qquad R_2 = \frac{4190785}{2048}, \qquad 2R_2 - D_2 = \frac{4183745}{2048}$$

6 Pinned certificate

The pinned verifier checks the exact Arthur-parameter normalization, Deligne bounds, character identities, multiplicative reserves, harmonic Rankin identities, an exact positive fixture, and a normalization counterfeit that must fail.

Run command

```
uv run --frozen python canon/witnesses/C-0018/verify.py
```

Pinned files

- `canon/witnesses/C-0018/PIN.md`
- `canon/witnesses/C-0018/verify.py`
- `canon/witnesses/K-0036/sources/chenevier-lannes-1409.7616v2.pdf`
- `canon/witnesses/K-0036/sources/deligne-modular-forms-l-adic-1969.pdf`
- `canon/witnesses/K-0036/sources/deligne-weil-i-1974.pdf`
- `canon/claims/C-0018-arthur-harmonic-packet-cone.md`

7 Scope

The inequalities concern exact Dirichlet coefficients in the absolute-convergence Euler-product normalization for the two stated Arthur parameters.

8 Sources

- Canonical claim: `canon/claims/C-0018-arthur-harmonic-packet-cone.md`
- Witness pin: `canon/witnesses/C-0018/PIN.md`
- Arthur parameter source: `canon/witnesses/K-0036/sources/chenevier-lannes-1409.7616v2.pdf`

- Ramanujan source chain: `canon/witnesses/K-0036/sources/deligne-modular-forms-l-adic-1969.pdf`