Completed Weil positivity through support log 7

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2026-07-18

Abstract

The completed centered Weil quadratic form is strictly positive for every nonzero real even test function supported in $[-R, R]$ when $R \le (\log 7)/2$. The proof identifies the form with a finite-source radial continuum operator, certifies positivity at the chamber endpoints, and transports the endpoint bounds to smaller supports by an exact dilation. A related exact matrix family is uniformly eventually positive on a shorter parameter interval.

$$\mathcal{Q}_\xi(\phi) > 0 \quad \big(0 \ne \phi \text{ real even, } \operatorname{supp}\phi \subset [-R, R],\ R \le \tfrac{1}{2}\log 7\big)$$

1 What is proved

Write $q = 4/t$ for the autocorrelation-support parameter. The complete radial continuum form $\mathfrak{A}_t^{\mathrm{full}}$ is strictly positive for every $t \ge 4/\log 7$, which is equivalent to completed centered Weil positivity for real even tests supported in $[-R, R]$ with $R \le (\log 7)/2$.

For the exact Jacobi-whitened matrices $W_n(tn)$, there is also a uniform eventual-positivity theorem for $t \in [4/\log 6, 7.2056493]$. Every fixed ray $t \ge 4/\log 2$ is eventually positive as well.

$$\mathfrak{A}_t^{\mathrm{full}} \succ 0 \qquad \left(t \ge \frac{4}{\log 7}\right)$$

$$\exists N\ \forall n \ge N\ \forall t \in \left[\frac{4}{\log 6}, 7.2056493\right], \quad W_n(tn) \succ 0$$

2 The Weil-operator dictionary

The Euler-free operator $\mathfrak{A}_t^+$ is corrected by the finitely many prime-power translations whose logarithms lie below $q = 4/t$. With $b_d = \Lambda(d)/(2\sqrt{d})$ and $a_d(t) = t\log d/2$, the active terms are compressed even translations $C_{a_d(t)}$.

An exact intertwining identity identifies this radial form with one half of the completed centered Weil form. Thus the operator inequality is not an analogy or a Galerkin surrogate: it is the bounded-support Weil statement itself.

$$\mathfrak{A}_t^{\mathrm{full}} = \mathfrak{A}_t^{+} - \sum_{\log d < 4/t} \frac{\Lambda(d)}{2\sqrt{d}}\, C_{t\log(d)/2}$$

$$\mathfrak{A}_t^{\mathrm{full}}(f) = \tfrac{1}{2}\mathcal{Q}_{\xi}(\phi_{t,f})$$

3 Endpoint certificates and chamber closure

At the endpoint $q = \log 7$, the active atoms are $2, 3, 4, 5$. Their exact piecewise action on shifted-Legendre modes is used to form the head matrix, the head-to-tail Gram term, and a Feshbach-Schur complement. High-precision interval congruence proves a positive tail floor and a positive finite Schur matrix.

The same architecture supplies endpoint floors for the preceding atom chambers. An exact cutoff-subspace dilation embeds every smaller support into the endpoint space, so the min-max principle transports each endpoint eigenvalue floor across its entire chamber.

$$(J_{q,Q}f)(x) = \sqrt{Q/q}\, f(Qx/q)\mathbf{1}_{(0,q/Q)}(x)$$

$$\mathfrak{A}_{4/q}^{\mathrm{full}}(f) = \mathfrak{A}_{4/Q}^{\mathrm{full}}(J_{q,Q}f), \qquad q \le Q$$

1. Represent each endpoint operator by exact action and moment data on the cells cut by support and reflection walls.
2. Lower-bound the omitted high modes and certify the finite Schur complement by directed Arb arithmetic.
3. Use the exact dilation identity and min-max to inherit positivity for every smaller support in the chamber.

4 Transfer to the finite matrices

Each Euler atom in $W_n(r)$ is an exact half-integer cosine compression. After adjacent parity pairing, the alias branch is suppressed on the critical frequency scale, while logarithmic-frequency compactness sends the primary branch to the corresponding continuum translation.

Mosco compactness, aggregate control of later atoms, and a parity-minus Schur floor tending like $\frac{1}{2}\log n - O(1)$ transfer the continuum gaps uniformly through the chambers ending at $q = \log 6$. The argument proves existence of a threshold N but extracts no numerical value for it.

5 What the pinned certificate checks

The public command runs the pinned outer-chamber certificate only. It covers $t \in [4/\log 2, 7.2056493]$ by 424 adjacent rational interval cells and proves, on every cell, a positive tail floor, positive Schur determinant, positive congruence pivots, and nonzero preconditioner determinants.

This certificate is one of six chamber certificates used by the canonical claim. The log-3 through log-7 endpoint certificates, nesting arguments, and finite-transfer proofs are separate named source files in the claim record; the pinned outer-chamber run is a reproducible anchor, not by itself a certificate of every assertion above.

$$\tau_t > 1.3514852913, \qquad \det S_t > 1.1750152695 \times 10^{-3}$$

6 Pinned certificate

The pinned 512-bit Arb program certifies the scalar-plus outer chamber uniformly on 424 exact interval cells. It supplies one load-bearing chamber anchor; the canonical theorem also relies on separately named endpoint, nesting, and finite-transfer certificates.

Run command

```
uv run --frozen python canon/witnesses/C-0007/certify_interval_operator.py
```

Pinned files

- `canon/witnesses/C-0007/certify_interval_operator.py`
- `canon/witnesses/C-0007/PIN.md`
- `canon/witnesses/C-0007/interval_operator_certificate.json`
- `canon/claims/C-0007-chamber-ladder-certified-range.md`

7 Scope

Continuum positivity is established through autocorrelation support $\log 7$; uniform eventual finite-matrix positivity is established through $\log 6$ with a non-effective threshold.

8 Sources

- Canonical claim: `canon/claims/C-0007-chamber-ladder-certified-range.md`
- Pinned certificate description: `canon/witnesses/C-0007/PIN.md`